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Abstract. We have carried out an elaborate study of electrical conduction in the generalized Thue–Morse (GTM) lattice. We have studied (i) the Landauer resistance, trace map and localization length of GTM structures, and (ii) the effects of deviations of inter-barrier distances from ideal GTM structures on electrical conduction. Among other things, our results indicate clearly the conditions under which a GTM lattice is likely to be most akin to a periodic system.

1. Introduction

During the past 15 years or so, many researchers have been engaged with the study of the electronic properties of one-dimensional (1D) quasiperiodic (QP) systems. In terms of its degree of periodicity, a QP system may be regarded as being intermediate between a periodic system and a truly disordered system. Studies of 1D quasiperiodic systems concentrate primarily on (i) the Fibonacci lattice and (ii) the Thue–Morse (TM) lattice. Historically, the study of the Fibonacci lattice began earlier than that of the TM lattice, and several theoretical treatments regarding the electronic properties of this kind of lattice have now appeared in the literature [1–8]. Further, the Fibonacci lattice has also been realized experimentally [9] in the form of the GaAs–AlAs superlattice; in fact, this experimental work greatly stimulated the subsequent investigation of the Fibonacci lattice. The studies of Fibonacci lattices, like those in [1–9], stimulated interest in regard to what is known in the literature as the generalized Fibonacci (GF) lattice, and several authors [10–16] have treated the electronic properties of this kind of lattice. Theoretical treatments of the GF lattice indicated [11] that it is expected to exhibit richer physical properties than the Fibonacci lattice.

As mentioned before, the study of the TM lattice began later than that of the Fibonacci lattice. As far as we could see, the first work on the TM lattice is that due to Axel *et al* [17]. The intention behind the study of the TM lattice by these authors was to extend the knowledge of 1D QP systems beyond the realm of the Fibonacci lattice, and this intention was again motivated by the hope expressed by Levine and Steinhardt [18] for achieving a close link between disordered systems, quasiperiodic systems and periodic systems. The work of Axel *et al* [17] was followed by theoretical studies of the TM lattice by several others [19–24], as well as experimental realization of the TM lattice [25, 26]. One major objective behind many studies [15, 19, 24] of the TM lattice was to compare the degree of aperiodicity of the TM lattice with that of the Fibonacci lattice.

The studies of the TM lattice stimulated interest in the generalized Thue–Morse (GTM) lattice, in the same way as the studies of the Fibonacci lattice did in regard to the GF lattice. However, it appears that treatments [23, 27, 28] of the GTM lattice carried out so far have

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not taken adequate care of some issues. So, the purpose of this paper is to deal effectively with these issues, which are as indicated below.

(*a*) Knowledge of the Landauer resistance [29] of 1D QP systems and its linkage with the trace map [28] and localization length [30] are of great importance in regard to electrical conduction in such systems [24]. It appears that the study of these aspects in regard to the GTM lattice has received limited attention so far [23]. Consequently, we thought it worth while to make a thorough investigation of these aspects, especially in the context of the circumstances under which a GTM lattice is likely to be most akin to a periodic system.

(*b*) We have studied the effects of departure from ideal GTM structure on the relevant Landauer resistance. Our study of this issue is motivated by a similar one carried out previously by Das Sarma and Xie [8] in regard to the Fibonacci lattice. As pointed out by those authors, in any experimentally constructed quasiperiodic system, the inter-barrier distances will vary somewhat from the corresponding ones required by mathematical sequences concerned with them. As a result, some random disorder is inevitably introduced into the quasiperiodic structures fabricated experimentally. Das Sarma and Xie investigated the maximum variation of this sort that would preserve the qualitative nature of the transport properties of the Fibonacci lattice. We feel that an investigation of this kind is likely to be of great utility in regard to GTM lattices as well.

Our studies of the aspects under both (*a*) and (*b*) are carried out (i) for certain GTM sequences (including the ordinary TM lattice as a special case) corresponding to fixed number of barriers in the chain, and (ii) for an associated periodic system (APS) with the same number of barriers as in (i). As we shall see later, a comparative analysis of Landauer resistance (LR), trace map and localization length of the GTM lattices brings out information regarding relative degrees of aperiodicity of the GTM lattices that we have considered. Also, a comparative analysis of the effects of departures from ideal inter-barrier distances on the transport properties of these GTM lattices indicates clearly which kind of GTM lattice is most immune to such departures. Further, this comparative analysis indicates that the GTM lattice is capable of spanning systems ranging from quasiperiodic to periodic ones.

The essential features of GTM lattices, together with those of our model, are incorporated in section 2. The derivation of LR, trace map and localization length of the GTM lattice are given in section 4. The treatments of section 4 are dependent on certain features of transfer matrices for the Kronig–Penney (KP) model on aperiodic lattices, and these features are elucidated in section 3. Numerical analyses pertinent to our various analytical results are presented in section 5. Finally, a critical discussion of our results and relevant conclusions are presented in section 6.

2. Some essential features of the generalized Thue–Morse lattice and our model

We realize a GTM lattice by placing rectangular potential barriers of fixed height and width along a 1D two-tile aperiodic lattice corresponding to a GTM sequence. The separation between the centres of two consecutive barriers takes one of two values, c and d, of the tiles, which are arranged according to the GTM sequence. It may be noted that the procedure we have followed in constituting a GTM lattice amounts to realizing the KP model on a GTM sequence. Our treatment of electrical conduction through the GTM lattice is concerned with electrons having sub-barrier energy.

A GTM sequence, S_l, is defined as

$$S_{l+1} = \{S_l^n, \bar{S}_l^m\} \qquad l \geqslant 0 \qquad n, m \geqslant 1. \tag{1}$$

The 'ordinary' TM lattice corresponds to $m = n = 1$. The basic unit of the GTM is S_0, which is given by

$$S_0 = \{c, d\}. \tag{2}$$

The symbol $\{c, d\}$ means that c and d are arranged in that order. $\bar{S}_l$ is the complement of S_l, obtained by interchanging c and d. The total number of barriers, N_l, in a GTM lattice is given by

$$N_l = 2(m + n)^l. \tag{3}$$

3. Transfer matrices for the Kronig–Penney model on aperiodic lattices

For our treatment of LR and related issues of the GTM lattice, we need some features of transfer matrices relevant to the KP model on this kind of lattice. In this section, we discuss the transfer matrices generally in the context of an aperiodic lattice, and apply them subsequently to the case of the GTM lattice.

The Hamiltonian H for a system of N barriers, with their centres at x_j, is given by

$$H = -\frac{\hbar^2}{2m_0}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \sum_{j=1}^{N} V(x - x_j). \tag{4}$$

$V(x - x_j)$ takes the constant value V_0 for $(x_j - b/2) \leqslant x \leqslant (x_j + b/2)$, and it is zero elsewhere. The Hamiltonian given by (4) is a continuous Hamiltonian. This kind of Hamiltonian appears to be capable of taking care of realistic features better than the so-called discretized Hamiltonian [31], which uses tight-binding approximation.

Now, the Schrödinger equation for Hamiltonian H yields the following wavefunction, ψ_j, for the zero-potential region between the jth and $(j + 1)$th barriers in a chain of N barriers:

$$\psi_j = A_j \exp[\mathrm{i}k(x - x_j - b/2)] + B_j \exp[-\mathrm{i}k(x - x_j - b/2)] \tag{5}$$

where $k^2 = 2m_0E/\hbar^2$; E = energy eigenvalue of the electron.

We now introduce two (2×2) matrices, $\mathbf{M}_j$ and $\mathbf{M}^{(N)}$, defined as below:

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = \mathbf{M}_j \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} A_N \\ B_N \end{pmatrix} = \mathbf{M}^{(N)} \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} \tag{7}$$

where

$$M_j(11) = [M_j(22)]^* = \{\cosh(\beta b) + \mathrm{i}[(k^2 - \beta^2)/(2k\beta)]\sinh(\beta b)\}\exp[\mathrm{i}k(\Delta x_j - b)] \tag{8}$$

$$M_j(12) = [M_j(21)]^* = -\mathrm{i}[(k^2 + \beta^2)/(2k\beta)]\sinh(\beta b)\exp[-\mathrm{i}k(\Delta x_j - b)] \tag{9}$$

$$\det \mathbf{M}_j = 1 \tag{10}$$

$$\mathbf{M}^{(N)} = M_N \ldots M_1 \tag{11}$$

$$M^{(N)}(11) = [M^{(N)}(22)]^* \tag{12}$$

$$M^{(N)}(12) = [M^{(N)}(21)]^* \tag{13}$$

$$\det \mathbf{M}^{(N)} = 1 \tag{14}$$

and

$$\Delta x_j = (x_j - x_{j-1}) \qquad \beta^2 = 2m_0(V_0 - E)/\hbar^2 \qquad E < V_0.$$

If c and d are interchanged in the two-tile aperiodic sequence, the axis of symmetry of potential barriers is shifted to new positions $\bar{x}_j$; consequently, the matrices $\mathbf{M}_j$ and $\mathbf{M}^{(N)}$ change respectively to $\bar{\mathbf{M}}_j$ and $\bar{\mathbf{M}}^{(N)}$, as shown below:

$$\bar{\mathbf{M}}_j = \mathbf{M}_j \qquad \text{with } \Delta x_j = \Delta\bar{x}_j = (\bar{x}_j - \bar{x}_{j-1}) \tag{15}$$

$$\bar{\mathbf{M}}^{(N)} = \bar{M}_N \ldots \bar{M}_1. \tag{16}$$

4. Landauer resistance and related issues of the generalized Thue–Morse lattice

In this section, we incorporate the treatments of (i) trace map, (ii) Landauer resistance and (iii) localization length, with regard to the GTM lattice.

4.1. Trace map and related entities of GTM lattice

As mentioned earlier, the centres of the barriers in a GTM lattice are distributed according to the GTM sequence given by (1). The total number of barriers, N, in a GTM lattice then becomes a GTM number, N_l. Denoting now the matrices $\mathbf{M}^{(N)}$ and $\bar{\mathbf{M}}^{(N)}$ ($N = N_l$) by $\mathbf{G}_l$ and $\bar{\mathbf{G}}_l$ respectively, we can write the following recursion relations:

$$\bar{\mathbf{G}}_{l+1} = \bar{G}_l^m G_l^n \qquad l \geqslant 0 \tag{17}$$

$$\bar{\mathbf{G}}_{l+1} = G_l^m \bar{G}_l^n \qquad l \geqslant 0. \tag{18}$$

The initial conditions relevant to (17) and (18) are shown below:

$$\mathbf{G}_0 = M_2 M_1 \tag{19}$$

$$\bar{\mathbf{G}}_0 = M_1 M_2. \tag{20}$$

Applying trace commutative law to $\mathbf{G}_l$ and $\bar{\mathbf{G}}_l$, we have

$$R_l = \bar{R}_l \tag{21}$$

where

$$R_l = \tfrac{1}{2}\,\text{trace}\,\mathbf{G}_l \qquad \bar{R}_l = \tfrac{1}{2}\,\text{trace}\,\bar{\mathbf{G}}_l.$$

Using (17), (18) and (21), we can obtain the following results:

$$R_{l+1} = P_l U_{m-1}^{(l)} U_{n-1}^{(l)} - R_l U_{m-1}^{(l)} U_{n-1}^{(l)} Y_{mn}^{(l)} + U_{m-1}^{(l)} U_{n-1}^{(l)} Z_{mn}^{(l)} \tag{22}$$

where

$$P_l = U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}\{4R_lR_{l-1}^2 + [4R_{l-1}^2(R_{l-1}Y_{mn}^{(l-1)} - 1) - (4Z_{mn}^{(l-1)} - 1)]U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}$$
$$- 2R_{l-1}[2R_l + (2R_{l-1}Y_{mn}^{(l-1)} - 2Z_{mn}^{(l-1)} - 1)U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}]Y_{mn}^{(l-1)}$$
$$+ [R_l + U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}(R_{l-1}Y_{mn}^{(l-1)} - Z_{mn}^{(l-1)})][(Y_{mn}^{(l-1)})^2 - 2Z_{mn}^{(l-1)}]$$
$$-2R_{l-1}U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}W_{mn}^{(l-1)} + 2Z_{mn}^{(l-1)}[R_l + U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}(R_{l-1}(Y_{mn}^{(l-1)} + 2R_{l-1})$$
$$- (Z_{mn}^{(l-1)} + 1))] + U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}Z_{mn}^{(l-1)}\} \qquad (23)$$

$$W_{mn}^{(l)} = Z_{mn}^{(l)}Y_{mn}^{(l)} \qquad Z_{mn}^{(l)} = U_{m-2}^{(l)}U_{n-2}^{(l)}/U_{m-1}^{(l)}U_{n-1}^{(l)} \qquad Y_{mn}^{(l)} = U_{m-2}^{(l)}/U_{n-1}^{(l)} + U_{n-2}^{(l)}/U_{n-1}^{(l)}$$

$$U_n^{(l)} = \sin[(n+1)\cos^{-1}(R_l)]/\sin[\cos^{-1}(R_l)] \qquad \text{for } |R_l| < 1 \qquad (24)$$

$$U_n^{(l)} = \sinh[(n+1)\cosh^{-1}(R_l)]/\sinh[\cosh^{-1}(R_l)] \qquad \text{for } |R_l| \geqslant 1. \qquad (25)$$

Equation (22) is the two-dimensional trace map for the GTM lattice. This equation connects R_{l+1}, R_l and R_{l-1}. The values of R_0 and R_1 are obtained from their definitions. Using R_0 and R_1, one can obtain R_l from (22) for $l > 1$. When $m = n = 1$, equation (22) reduces to the dynamical trace map of the (ordinary) TM lattice:

$$R_{l+1} = 4R_{l-1}^2R_l - 4R_{l-1}^2 + 1 \qquad l \geqslant 1. \qquad (26)$$

The entities R_l and $\bar{R}_l$ are essentially the real parts of $G_l(11)$ and $\bar{G}_l(11)$ respectively. Consequently, we can write

$$G_l(11) = R_l + iI_l \qquad (27)$$

$$\bar{G}_l(11) = \bar{R}_l + i\bar{I}_l. \qquad (28)$$

It can be shown that I_l and $\bar{I}_l$ satisfy the following recursion relations:

$$I_{l+1} = F_lU_{m-1}^{(l)}U_{n-1}^{(l)} - I_lU_{m-1}^{(l)}U_{n-2}^{(l)} - \bar{I}_lU_{m-2}^{(l)}U_{n-2}^{(l)} \qquad (29)$$

$$\bar{I}_{l+1} = \bar{F}_lU_{m-1}^{(l)}U_{n-1}^{(l)} - \bar{I}_lU_{m-1}^{(l)}U_{n-2}^{(l)} - I_lU_{m-2}^{(l)}U_{n-1}^{(l)} \qquad (30)$$

where

$$F_l = U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}\{4R_lR_{l-1}I_{l-1} + 2R_{l-1}[I_{l-1}(2R_{l-1}Y_{mn}^{(l-1)} - 2Z_{mn}^{(l-1)} - 1) + \bar{I}_{l-1}]U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}$$
$$- 2R_{l-1}[U_{m-2}^{(l-1)}(\bar{I}_l + \bar{I}_{l-1}U_{m-1}^{(l-1)}U_{n-2}^{(l-1)} + I_{l-1}U_{m-2}^{(l-1)}U_{n-1}^{(l-1)})/U_{n-1}^{(l-1)}$$
$$+ U_{n-2}^{(l-1)}(I_l + I_{l-1}U_{m-1}^{(l-1)}U_{n-2}^{(l-1)} + \bar{I}_{l-1}U_{m-2}^{(l-1)}U_{n-1}^{(l-1)})/U_{n-1}^{(l-1)}]$$
$$+ (U_{m-2}^{(l-1)})^2(\bar{I}_l + \bar{I}_{l-1}U_{m-1}^{(l-1)}U_{n-2}^{(l-1)} + I_{l-1}U_{m-2}^{(l-1)}U_{n-1}^{(l-1)})/(U_{m-1}^{(l-1)})^2$$
$$+ (U_{n-2}^{(l-1)})^2(I_l + I_{l-1}U_{m-1}^{(l-1)}U_{n-2}^{(l-1)} + \bar{I}_{l-1}U_{m-2}^{(l-1)}U_{n-1}^{(l-1)})/(U_{n-1}^{(l-1)})^2$$
$$- 2I_{l-1}(R_l + R_{l-1}U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}Y_{mn}^{(l-1)} - U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}Z_{mn}^{(l-1)})Y_{mn}^{(l-1)}$$
$$+(I_{l-1} - \bar{I}_{l-1})U_{m-1}^{(l-1)}U_{n-1}^{(l-1)} + (I_{l-1} + \bar{I}_{l-1})U_{m-1}^{(l-1)}U_{n-1}^{(l-1)}(4R_{l-1}Z_{mn}^{(l-1)} - W_{mn}^{(l-1)})\}. \qquad (31)$$

$\bar{F}_l$ can be obtained from (31) by changing I_l and I_{l-1} to $\bar{I}_l$ and $\bar{I}_{l-1}$ respectively.

As we shall see later, the LR of the GTM lattice is involved with, among other things, real as well as imaginary parts of $G_l(11)$ and $\bar{G}_l(11)$. Values of I_0, $\bar{I}_0$, I_1 and $\bar{I}_1$ are obtained directly from the defining equations (27) and (28). Using these four entities, I_l and $\bar{I}_l$ can be conveniently determined from (29) and (30), for $l > 1$.

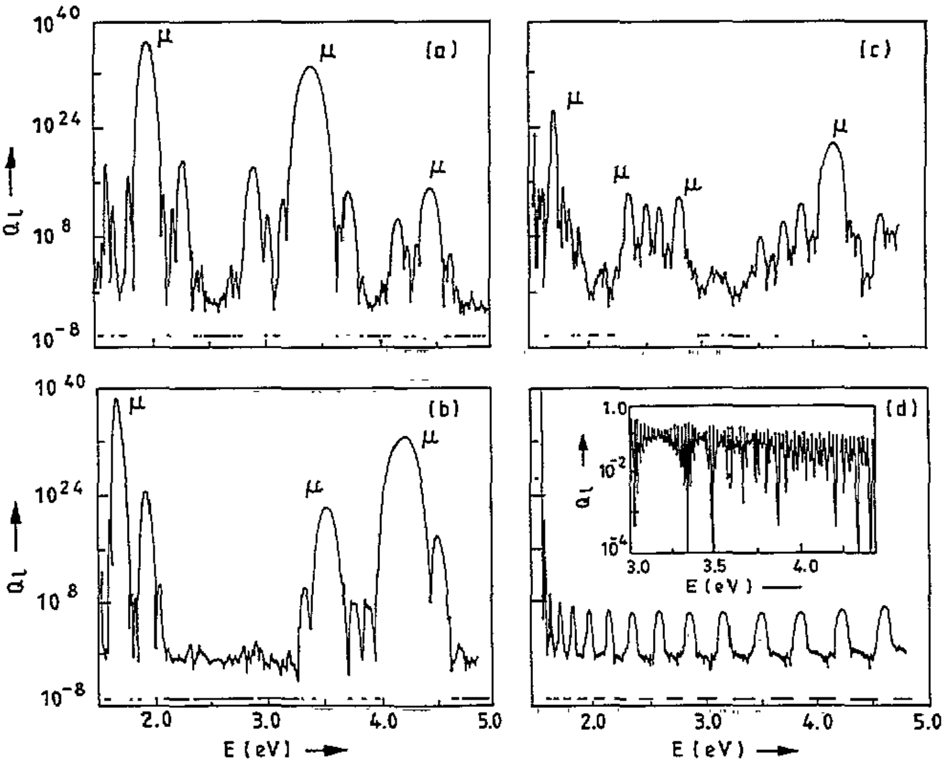

Figure 1. Variations of dimensionless resistance Q_l with energy (E) along with trace map. Parts (a), (b), (c), (d) refer respectively to GTM sequences with $(l, m, n) = (8, 1, 1)$, $(4, 2, 2)$, $(4, 3, 1)$, $(2, 15, 1)$. The trace maps for the corresponding cases are shown by horizontal shaded and unshaded regions, referring to allowed and forbidden regions of energy respectively.

4.2. Landauer resistance of GTM lattice

The Landauer resistance ρ of a chain of potential scatterers is given [29] by

$$\rho = (h/e^2)(1 - T)/T \tag{32}$$

where T is the transmission coefficient of the entire chain. Assuming that the electron is incident on the left end of the system, and taking note of the fact that there is no reflected part of the wavefunction beyond the right end of the chain, we get from (32) the following formula for the Landauer resistance ρ_l of a GTM lattice corresponding to GTM sequence S_l:

$$\rho_l = (h/e^2)Q_l \tag{33}$$

$$Q_l = |G_l(12)|^2. \tag{34}$$

The *dimensionless entity Q_l contains all essential information about* LR *and, henceforth, we* will consider Q_l as the effective LR for convenience [32]. Now noting that $\mathbf{G}_l = \mathbf{M}^{(N_l)}$, and taking account of (12)–(14), we can obtain from (34) the following form of Q_l:

$$Q_l = (R_l^2 + I_l^2) - 1. \tag{35}$$

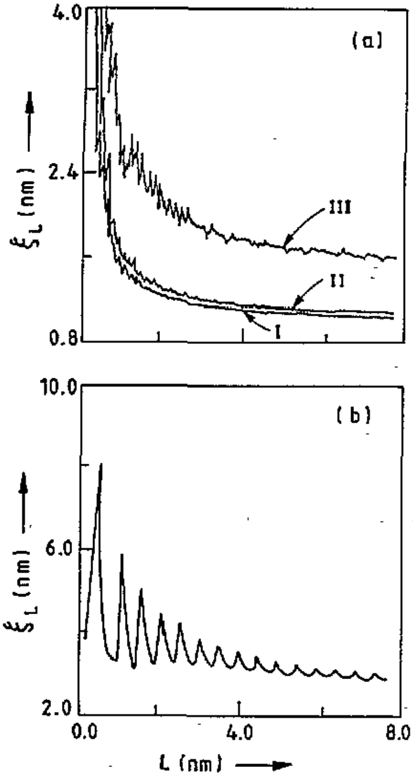

Figure 2. Variation of localization length ξ_L with length (L) of the sample. In (a), graphs I, II and III correspond respectively to $(l, m, n) = (8, 1, 1)$, $(4, 2, 2)$ and $(4, 3, 1)$. Part (b) corresponds to GTM sequence with $(l, m, n) = (2, 15, 1)$.

4.3. Localization length of GTM lattice

We have investigated the localization length ξ_L of a GTM lattice at zero temperature. ξ_L is obtained [30] from the following well known formula:

$$\xi_L = L/\ln(1 + \langle Q_l \rangle). \tag{36}$$

Here L is the length of the GTM chain and $\langle Q_l \rangle$ means the ensemble average of Q_l. To evaluate $\langle Q_l \rangle$ for our case, we have followed the procedure used by Das Sarma and Xie [8]; the details regarding this procedure are discussed later (section 5).

5. Numerical analyses

Our numerical studies are concerned with (i) variation of Q_l with energy, (ii) trace map of GTM lattice, (iii) variation of ξ_L with length (L) of GTM chain, and (iv) effects of deviations of values of c and d from ideal GTM sequence on Q_l and $\langle Q_l \rangle$. The results about aspects (i) and (ii) are obtained on the basis of (35) and (22) respectively, and they are shown in figure 1. As regards aspect (iii), we have used (36) and the results are shown in figure 2. To determine $\langle Q_l \rangle$ in (36), we taken an interval $k \in [k_0, k_0 + t\Delta k]$, and then define [8] the average resistance $\langle Q_l \rangle$ on the k-mesh as

$$\langle Q_l \rangle = \frac{1}{t+1} \sum_{i=0}^{t} Q_l(k_0 + i\Delta k). \tag{37}$$

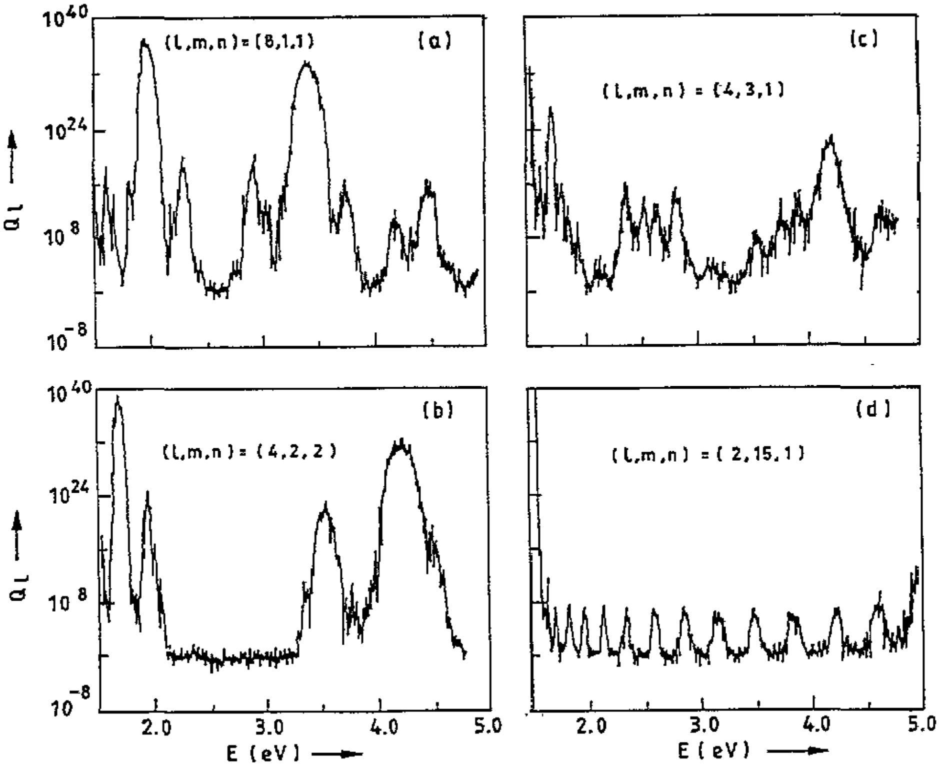

Figure 3. Variation in Q_l with E for non-ideal GTM lattices characterized by 5% deviation in values of c and d of ideal GTM sequence.

We have taken the interval of k as [8.0, 11.0] nm^{-1} and $\Delta k = 0.001\ \mathrm{nm}^{-1}$.

For aspects (i)–(iii), we have assumed the total number of barriers fixed at 512. For such a case, we get four GTM lattices with $(l, m, n) = (8, 1, 1)$, $(4, 2, 2)$, $(4, 3, 1)$ and $(2, 15, 1)$; it may be noted that the case $(8, 1, 1)$ represents the (ordinary) TM lattice with 512 barriers. As regards values of parameters concerned with aspects (i)–(iii), we have taken the following ones for all these cases: $V_0 = 5$ eV, $c = 0.1$ nm, $d = 0.2$ nm and $b = 0.05$ nm.

The values of Q_l and $\langle Q_l \rangle$ for non-ideal GTM lattices (aspect (iv)) are obtained respectively from (35) and (37), by introducing variations in the values of c and d and keeping all other parameters the same as those in respect of (i)–(iii). The results about Q_l of non-ideal GTM lattices are shown in figures 3 and 4, while the corresponding results for $\langle Q_l \rangle$ are given in table 1.

6. Results, discussion and conclusions

The results shown in figures 1–4 and table 1 provide us with many important features of electrical conduction in GTM lattices constructed out of a fixed number (512) of barriers. We now elucidate these features, which together indicate, among other things, what kind of GTM lattice is likely to be most akin to a periodic system.

Looking at figure 1, we note the following features:

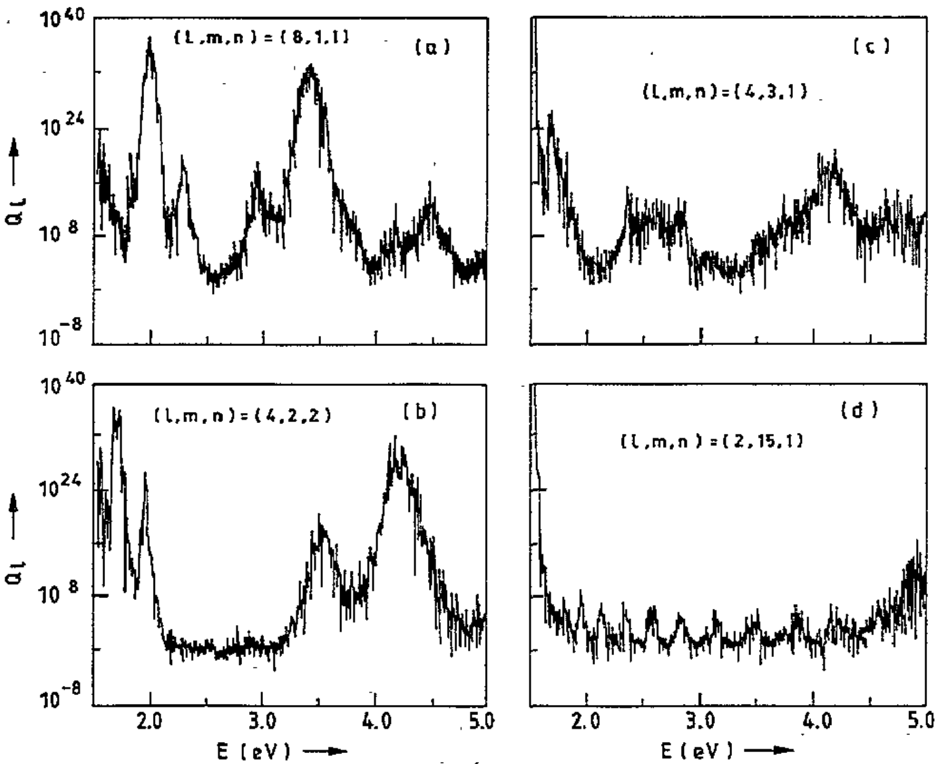

Figure 4. Variation of Q_l with E for non-ideal GTM lattices with 10% deviation in values of c and d of ideal GTM lattices.

(i) For all varieties of GTM lattice, including the special case of the ordinary TM lattice, Q_l shows oscillations with energy. The values of Q_l are large in the gap regions and small in the allowed regions of the relevant trace maps.

(ii) When the ratio m/n is not much different from unity (graphs (*a*)–(*c*) in figure 1), the oscillations of Q_l with energy are involved with two kinds of maxima. Maxima of one kind (marked by μ) are much larger than maxima of the other kind, which occur on both sides of the larger ones. We call the former class 'principal maxima' and the latter class 'secondary maxima' [24].

(iii) The graph (*d*) in figure 1 shows that, when m/n becomes quite large compared to unity, the oscillations of Q_l resemble those of LR of a periodic system. The oscillations of LR of a periodic system with 512 barriers are shown in the inset of (*d*); these oscillations are seen to be much more rapid than those of Q_l. We showed previously [24] that the oscillations of LR of a periodic system with energy become less rapid with decrease of number of barriers in the system. Considering together this fact and the graphs in (*d*), we can conclude that, for large m/n, the GTM lattice resembles a periodic system with fewer barriers than that for the GTM lattice under consideration. In other words, we may say that, for a large value of m/n, $N_p/N_l < 1$, where N_l is the number of barriers in the GTM lattice with large value of m/n and N_p is the number of barriers in the periodic system that is equivalent to this GTM lattice.

We now come to discuss the features of ξ_L yielded by the graphs in figure 2. These

Table 1. Average resistance $\langle Q_l \rangle$ taken over the k-mesh from $k = 8.0$ to $11.0\ \text{nm}^{-1}$.

l	m	n	Deviations of c and d (%)	$\langle Q_l \rangle$
8	1	1	0.0	8.39×10^{31}
8	1	1	5.0	5.73×10^{31}
8	1	1	10.0	5.18×10^{30}
8	1	1	15.0	4.12×10^{28}
4	2	2	0.0	6.61×10^{30}
4	2	2	5.0	4.83×10^{30}
4	2	2	10.0	1.03×10^{30}
4	2	2	15.0	2.43×10^{28}
4	3	1	0.0	4.26×10^{20}
4	3	1	5.0	2.70×10^{20}
4	3	1	10.0	9.19×10^{19}
4	3	1	15.0	2.11×10^{19}
2	15	1	0.0	2.84×10^{11}
2	15	1	5.0	5.40×10^{12}
2	15	1	10.0	9.55×10^{16}
2	15	1	15.0	4.29×10^{20}

graphs show that, although ξ_L exhibits oscillations for small values of L, it becomes almost independent of L for large values of L. This situation indicates that Anderson localization [33] occurs for all the cases shown in figure 2; further, this situation is in conformity with the concept that, in 1D systems, all eigenstates are localized for any type of disorder [34]. However, it may be noted that the value of ξ_∞, which is the value ξ_L assumes for large L, depends on the extent of disorder. For (2,15,1) GTM chain, $\xi_\infty \sim 3.0$ nm; for (8, 1, 1) GTM chain, which is the ordinary TM chain, $\xi_\infty \sim 1.0$ nm. This observation shows that, for large m/n value, ξ_∞ is large compared to what it is when m/n is nearly unity. Since a large value of ξ_L implies high degree of periodicity of the lattice, we may conclude that GTM lattices with larger values of m/n are more akin to a periodic system than those having m/n nearly equal to unity. This conclusion is in conformity with what we discussed earlier in connection with figure 1.

To study the effects of deviations from ideal GTM structures on electrical conduction, we have introduced extra disorder by making the inter-barrier distances c and d vary randomly from layer to layer in the GTM structures. Thus, we have taken $c = c_0 + \Delta c$ and $d = d_0 + \Delta d$, where $c_0 = 0.1$ nm and $d_0 = 0.2$ nm, Δc and Δd being the random disorder in c and d respectively. The graphs in figure 3 show that 5% variations in c and d do not introduce any substantial change in Q_l of any GTM lattice. When we go to 10% variations in c and d (graphs in figure 4), Q_l for (2, 15, 1) case seems to be affected much more than the other three cases. This observation is indicated more clearly by the results of table 1. Looking at this table, one can see that the order of $\langle Q_l \rangle$ remains unaffected up to 10% variation in c and d, for all cases except the (2, 15, 1) GTM lattice. Thus, one can say that GTM lattices with high m/n value are most sensitive to variations in values of c and d and, conversely, GTM lattices with m/n nearly equal to unity are least sensitive to such variations. Now, we discussed previously that a large value of m/n of a GTM lattice makes it very much akin to a periodic system. This fact, together with information we obtained from figure 3 and 4 and table 1, indicates that the more akin a GTM lattice is to a periodic system, the more sensitive it is to variations in c and d. Further, our results in figures 3 and 4 and table 1 also seem to indicate a threshold behaviour of the effect of disorder in c and d on transmission properties.

As mentioned before, in practical realization of any quasiperiodic system, deviations from ideally desired quasiperiodic sequence are unavoidable due to technical limitations. Hence, some knowledge about the sensitivity of the desired systems to variations in parameters like inter-barrier distances are likely to be of great help in practice. In this paper, we have carried out an elaborate study of the issue with regard to GTM lattices, the motivation of this study being provided by a previous work of this kind with regard to Fibonacci lattices [8]; we feel that treatment of this issue for all other types of quasiperiodic systems would also be of great practical importance.

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